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AEROPLACE ASSEMBLY SENSITIVITY ANALYSIS BASED ON STATE SPACE MODEL

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ABSTRACT

The concept of Key Control Characteristics (KCC) and Key Product Characteristics (KPC) are presented in this article. Based on this, assembly sensitivity is defined, and it is described in feature, stage and system level. On the basis of state space model of assembly process, this paper considers the problem of assembly sensitivity analysis in a multi-station assembly process. Based on the concept of Key Control Characteristics (KCC) and Key Product Characteristics (KPC), we focus on the unique challenges brought by the multi-station system, namely, a system level model to characterize the variation propagation in the entire process, and the necessity to describe the system response to variation inputs at both global (system level) and local (station level and single fixture level) scales. State space representation is employed to recursively describe the propagation of variation in such a multi-station process, incorporating process design information such as fixture locating layout at individual stations and station-to-station locating layout change. Following the sensitivity analysis in control theory, an aeorplane assembly example is analyzed.

KEYWORDS

Assembly sensitivity, State space model, Aeroplanes, key product characteristics and Key control characteristics.

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INTRODUCTION

Structures of aeroplanes are becoming more and more complicated nowadays. The number of cabins has also increased. The angular variation is an important as well as rigorous index in terms of the assembly of columned parts, which has become more obvious because of assembly variation propagation. In this sense, the assembly angular variation should be reduced for the sake of coherence, stability and reliability of aeroplanes. Since different point in assembly process makes different influence on the assembly accuracy, namely, the assembly sensitivity is different, which causes different difficulty on assembly accuracy control. So, sensitivity analysis in assembly process is important to reduce the assembly costs.

Assembly sensitivity analysis is one of the most important basis of assembly accuracy prediction and controlling. However, relevant research is limited. LIU Y. S[1] *et al.* discussed the vector model of deviation, ANSELMETTI and LIU W. D[2,3]. presented the concepts of variation sources, analyzed the variation propagation mechanism, but they ignored the influence of assembly sensitivity on variation propagation in three dimension. LIU[4] *et al*. extended the variation description to three dimensions, and they built the state space model of three dimensional variation propagation. However, they only regarded the positional variation as the variation source. LI[5] *et al*. presented a tolerance analysis method based on assembly features, which mixes positional variation with fixture errors. QURESHI[6] *et al.* described the assembly variation as a kind of clearance in the analyzing process, which is good to the tolerance analysis with interference fit. ANSELMETTI[7] built the 'Jacobi-vector pair' model, which realizes the combination of Jocabi matrix and vector pair methods. HUANG[8] *et al*. built a multi-station assembly variation propagation model based on 3-2-1 fixture, and a three dimensional assembly model is established.

The sensitivity analysis of multi-station processes is under researched mainly due to the unavailability of a system level model which could link the KCC's variation to the KPC's quality. The challenges are also caused by the requirement of having comprehensive benchmarking at the system level, the station level, and a single KCC (fixture) level, as discussed in Section 2. Very few research papers were published in this area except for the paper of Suri and Otto[11], which developed an Integrated System Model (ISM) for stretch forming process. They used a linearized predictive variation model integrated with an FEM (Finite Elements Method) models for stretch forming and heat treatment processes.

ASSEMBLY VARIATION SENSITIVITY BASED ON STATE SPACE MODEL

Product quality is characterized by a group of features that could greatly affect the designed functionality and the level of customer satisfaction. In the automotive industry, this group of critical features is known as KPC (Key Product Characteristics). The fixture locators are the dimensional control characteristics for product positioning and thus are the determining factors in achieving the required dimensional accuracy of KPCs. They are known as KCCs (Key Control Characteristics). In a multi-station process, the impact of KCCs' variation on KPC's dimensional accuracy depends on process design configuration including the geometry of fixture locating layout on every station and the station-to station locating layout change. Early design evaluation of multistation assembly processes is very important for new product development and also for designing a robust manufacturing system to improve product quality.

The basic idea of developing the state space model is to consider the multi-station process as a sequential dynamic system but replace the time index in the traditional state space model with a station index. The state space model includes two equations:

After taking part and fixture error and reorientation-induced deviation into consideration, the stream- of-variation model can be described in Figure No.5.

The stream-of-variation model can be characterized by the following equations:

$$\mathbf{X}(i) = \mathbf{A}(i-1)\mathbf{X}(i-1) + \mathbf{B}(i)\mathbf{U}(i) + \mathbf{W}(i) \qquad i = 1, 2, \ldots, N \qquad (1)$$

$$\mathbf{Y}(i) = \mathbf{C}(i)\mathbf{X}(i) + \mathbf{V}(i) \qquad i = 1, 2, \ldots, N \qquad (2)$$

System matrices A, B, and C are determined by the process/product design. Matrix A, known as the dynamic matrix, characterizes the assembly reorientation during part transfer between stations. In other words, A depends on the station-to-station locating layout change in a production stream. Matrix B is the input matrix which determines how the fixture deviation affects part deviation, depending on the geometry of a fixture layout. Matrix C contains the information about sensor positions on product, which are often the selected KPC points during design stage. The index of the observation equation Eq.2 is normally a subset of {1, 2, N} since KPCs are not measured on all stations. Usually, KPCs are selected on the final product in a design problem, and thereby, we can obtain the following input-output relationship:

$$Y=\sum_{k=1}^{N}\gamma(k)U(k)+\gamma(0)X(0)+\varepsilon \quad (3)$$

Where

$$\gamma(k)=C\Phi(N,k)B(k) \text{ And } \gamma(0)=C\Phi(N,0) \quad (4)$$

$$\varepsilon=\sum_{k=1}^{N}C\Phi(N,k)\xi(k)+\eta \quad (5)$$

The input-output covariance relationship could be obtained from Eq.3

$$K_Y=\sum_{k=1}^{N}\gamma(k)K_U(k)\gamma^T(k)+\gamma(0)K_0\gamma^T(0)+K_\varepsilon$$
$$K_Y=\sum_{k=1}^{N}\gamma(k)K_U(k)\gamma^T(k) \quad (6)$$

This relationship expresses the variation of KPCs (K_Y) in terms of the variation of KCCs ($K_P(k)$) at all stations, the part stamping variation (K_0), and the noise variation (K_ε). Since the goal of this paper is to benchmark fixture design configuration, we will focus our analysis on the variation of KCCs ($K_P(k)$). The impact of part variation and noise variation is not discussed in this paper. Therefore, we can simplify Eq. 6 and only keep terms related to KCCs' variation

$$K_Y=\sum_{k=1}^{N}\gamma(k)K_U(k)\gamma^T(k) \quad (7)$$

The proposed sensitivity analysis for design evaluation defines: how the system responds to certain variation inputs, which variation source contributes most to the final product variation, and/or ~3! how the process parameters account most for the variation propagation. As such, the sensitivity indices are similar to the system gains in the conventional control theory. Appropriate measure is introduced to represent the process sensitivity as the gain of a Multiple-Input-Multiple-Output (MIMO) system. Three-level sensitivity indices are defined to facilitate the description of the system behavior of a multi-station assembly process: single fixture level, station level with multi-fixture, and system level (multi-station).

The sensitivity-based design evaluation index at the fixture level, denoted as S_{kp}, is defined as

$$S_{ip}=\sup_{\sigma_{ip}^2}\frac{\left\|\kappa\sigma_{\text{out put}}^2\right\|_2}{\sigma_{ip}^2} \quad (8)$$

where weighting coefficient W determines the relative importance of KPC variances and $\|\cdot\|_2$ is the Euclidean norm. S_{kp} index indicates how the pth locating feature at station k contributes to the KPC variations. At this level, S_{kp} in fact, corresponds to the gain of a Single-Input-Multiple-Output (SIMO) system.

The sensitivity-based design evaluation index at the station level, denoted as S_k, is defined as

$$S_i=\sup_{\sigma_i^2}\frac{\left\|\kappa\sigma_{output}^2\right\|_2}{\left\|\sigma_i^2\right\|_2} \quad (9)$$

S_k index indicates how the fixture elements on station k jointly affect the KPC variation. It is a MIMO-type gain since each station contains multiple fixtures. Station-level sensitivity index S_k identifies the critical station contributing most to the KPC variation.

The sensitivity-based design evaluation index at the system level, denoted as S_o, is defined as

$$S_o=\sup_{\sigma_{input}^2}\frac{\left\|\kappa\sigma_{output}^2\right\|_2}{\left\|\sigma_{input}^2\right\|_2} \quad (10)$$

S_o index indicates the system capacity to amplify or suppress the input variations. S_o index is also a MIMO-type gain.

The above defined indices S_{kp}, S_k, and S_o are the ratios of the KPC variation over the KCC variation. Consider $\left\|\kappa\sigma_{\text{out put}}^2\right\|_2$ as the indicator of the KPC variation level. Indices S_{kp}, S_k, and S_o are the values of KPC variation given a unit KCC variation input. The unit of KCC variation is different for the three indices: for a single fixture, a unit KCC variation is equivalent to σ_{ip}^2 =1; for a station, a unit KCC variation is the joint effect from the multiple fixtures, defined as $\left\|\sigma_i^2\right\|_2$ =1; for the entire system, a unit KCC variation is the combining effect from the multiple stations, defined as $\left\|\sigma_{input}^2\right\|_2$ =1. A sensitivity index less than 1 means that the KPC variation level can become lower than the KCC variation level. On the contrary, a sensitivity index larger than 1 implies that the system amplifies the input variation. Most of the multi-station systems will end up with the sensitivity greater than 1. Nonetheless, the smaller sensitivity value suggests a less variationsensitive system which is preferable. Therefore, using and comparing this group of indices, a robust process configuration can be selected, and the sensitive station and fixture can be identified and prioritized.

Next, the indices S_{kp}, S_k, and S_o are expressed in terms of the model matrix γ so that they are made input-independent. If the KCC variation inputs in Fig.1 are uncorrelated, the KPC variance vector σ_{output}^2 can be represented as a linear combination of the vector σ_i^2. The expression can be represented as

$$\sigma_{output}^2 = \sum_{i=1}^{N}\left[\gamma^2(i)\right]\cdot\sigma_i^2 \tag{11}$$

where $\left[\gamma^2(i)\right]$ represents a matrix in which each element is the square of the corresponding element in matrix $\left[\gamma(i)\right]$, i.e.,

$$\left[\gamma^2\right] = \begin{bmatrix} \gamma_{11}^2 & \gamma_{12}^2 & \cdots & \gamma_{1m}^2 \\ \gamma_{21}^2 & \gamma_{22}^2 & \cdots & \gamma_{2m}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{q1}^2 & \gamma_{q2}^2 & \cdots & \gamma_{qm}^2 \end{bmatrix} \tag{12}$$

According to the definition of S_{kp} index, it is assumed that there is only a single variation source (rather than multiple simultaneous sources) in the entire process at each time. The fixture-level sensitivity index S_{kp} on station k can be expressed as

$$S_{ip} = \left\|\kappa\gamma_p^2(i)\right\|_2 \tag{13}$$

The second index is the station sensitivity index S_k. It is assumed that only one station has variation inputs at a time. But within each station, more than one fixture element could contribute to σ_{output}^2 simultaneously. The station-level sensitivity index S_k can be expressed as

$$S_k = \left\|\kappa\gamma^2(i)\right\|_2 \tag{14}$$

System-level sensitivity will consider all possible combinations of multiple KCC variation inputs-within a station and/or cross stations. Thus, it represents the overall sensitivity level of a process as to the KCC variation inputs. The system-level sensitivity index So can be expressed as

$$S_o = \left\|\kappa\cdot\left[\gamma^2(1) \quad \gamma^2(2) \quad \ldots \quad \gamma^2(N)\right]\right\|_2 \tag{15}$$

It is also possible to define the station and system sensitivity indices using the fixture sensitivity index, that is, choosing the largest fixture sensitivity index within a station or in a process as the station and system indices, respectively. Under this definition, these new indices could represent process response to a single variation input, whereas the proposed indices in this paper (Eq.9) and (Eq.10) describe the joint effect of multiple simultaneous variation inputs. The results are different using the two sets of definitions. The selection between both sets of indices depends on the specific requirements of applications.

ASSEMBLY SENSITIVITY CALCULATION OF AEROPLANES

First, KCC points P_1-P_6 in assembly process are defined. The coordinate values of these KCC points and KPC points are listed in **Error! Reference source not found.** and Table No.2: . The probable fixture sequence is shown in Table No.3.

According to the fixture sequences above, the state space model in Eq.1 is adopted. And all the KPCs are treated equally. So, all the weighting coefficients are set as 1.

In this assembly process, there are 2 assembly station and 1 measuring station, namely, N=3. Because the effect of fixture should be excluded in measuring station, the fixture error is far less than other stations, we consider it as zero in this article. The fixture errors in the other two stations are *U (1)* and *U (2)*. In this example, the angular variation in X axis is defined as the assembly accuracy, and the 2nd assembly sequence is adopted. The assembly process is shown in Figure No.2. The state space model of this assembly process is

$$\begin{cases} \mathbf{X}(1) = \mathbf{B}(1)\mathbf{U}(1) \\ \mathbf{X}(2) = \mathbf{A}(1)\mathbf{X}(1) + \mathbf{B}(2)\mathbf{U}(2) \\ \mathbf{X}(3) = \mathbf{A}(2)\mathbf{X}(2) \\ \mathbf{Y} = \mathbf{C}\mathbf{X}(3) \end{cases} \quad (16)$$

where A, B and C can be calculated according to the parameters in **Error! Reference source not found.**and Table No.2: . Eq.16 can be written as

$$\begin{aligned} \mathbf{Y} &= \mathbf{C}\left[\mathbf{A}(2)\left(\mathbf{A}(1)\mathbf{B}(1)\mathbf{U}(1) + \mathbf{B}(2)\mathbf{U}(2)\right)\right] \\ &= \mathbf{CA}(2)\mathbf{A}(1)\mathbf{B}(1)\mathbf{U}(1) + \mathbf{CA}(2)\mathbf{B}(2)\mathbf{U}(2) \end{aligned} \quad (17)$$

So $\gamma(1) = \mathbf{CA}(2)\mathbf{A}(1)\mathbf{B}(1)$, $\gamma(1) = \mathbf{C}\mathbf{A}(2)\mathbf{B}(2)$.

Next, the assembly sensitivity indices at station level can be calculated according to Eq.9. The results are in Table No.4: Then, based on Eq.18, the percentage of each station can be obtained.

$$PC_k = \frac{S_k}{\sum_{k=1}^{2} S_k}\% \quad (18)$$

At last, the indices of assembly sensitivity at fixture level can be calculated according to Eq.8. There are two independent mating features in each station. The fixture level assembly sensitivity is shown in Table No.5: .

ASSEMBLY SENSITIVITY ANALYSIS BASED ON MONTE CARLO METHOD

The analyzed aeroplane model is the same as the one in section 3. It also adopts the 3-2-1 fixing method.

The form and location tolerance of the setting model is shown in Table No.6: . The flatness and perpendicularity of every mating feature plane are all 0.01mm. And flatness and perpendicularity are linear tolerance. If the mating feature planes are considered as a plenty of discrete points, then the form and location tolerance of mating feature planes can be equivalent to the deviation of the nominal position of discrete points. As shown in Figure No.4.

So selecting some discrete points on mating feature plane to express the form and location tolerance. Setting the deviation of the points in the Normal distribution N(0, 0.01).

After 2000 simulation, the results are shown in Figure No.4.

We can find from the analysis results that the front face of B cabin has the lest influence rate 16.09%; the next one is the back face of A cabin, and the assembly sensitivity is 17.03%; the influence rate of the back face of B cabin can reach 33.16%; The greatest impact mating feature plane on assembly results is the front face of C cabin, namely, 33.72%. The analyzed results can be seen in Error! Reference source not found..

By comparison with Table No.5: and **Error! Reference source not found.**, the results show that the deviation between the 4 mating feature planes of the theoretical calculation and Monte Carlo simulation is 6.054%.

Table No.1: The coordinate values of KCC points

S.No	P_1	P_2	P_3	P_4	P_5	P_6
1	(240, 135, 0)	(240, -135, 0)	(240, -90.7, -100)	(540, 135, 0)	(540, -135, 0)	(540, 125.4,50)

Table No.2: The coordinate values of KPC points

S.No	M_1	M_2	M_3	M_4
1	(0, 95, 0)	(0, -95, 0)	(840, 135, 0)	(840, -135, 0)

Table No.3: The probable fixture sequence

S.No	Fixture sequence
1	【（P_1、P_2、P_3）$_{A\ cabin}$】$_{I}$→【（P_4、P_5、P_6）$_{B\ cabin}$】$_{II}$ →【(M_1、M_2)，（M_3、M_4）】$_{III}$
2	【（P_1、P_2、P_3）$_{B\ cabin}$】$_{I}$→【（P_4、P_5、P_6）$_{B\ cabin}$】$_{II}$ →【(M_1、M_2)，（M_3、M_4）】$_{III}$
3	【（P_4、P_5、P_6）$_{C\ cabin}$】$_{I}$→【（P_1、P_2、P_3）$_{B\ cabin}$】$_{II}$ →【(M_1、M_2)，（M_3、M_4）】$_{III}$

Table No.4: The assembly sensitivity indices at station level and their percentage

S.No		Station 1	Station 2
1	S_k	2.38	8.215
2	PC_k	22.4633%	77.5367%

Table No.5: The assembly sensitivity indices at fixture level

S.No		Station 1		Station 2
1	The back face of A cabin	15.972%	The back face of B cabin	34.028%
2	The front face of B cabin	15.972%	The front face of C cabin	34.028%

Table No.6: The form and location tolerance of mating feature planes

S.No		A-@front	A-@back	B-@front	B-@back	C-@front	C-@back
1	Flatness	0.01	0.01	0.01	0.01	0.01	0.01
2	Perpendicularity	0.01	0.01	0.01	0.01	0.01	0.01

Table No.7: The assembly sensitivity at fixture level by Monte Carlo simulation

S.No		Station 1		Station 2
1	The back face of A cabin	17.03%	The back face of B cabin	33.16%
2	The front face of B cabin	16.09%	The front face of C cabin	33.72%

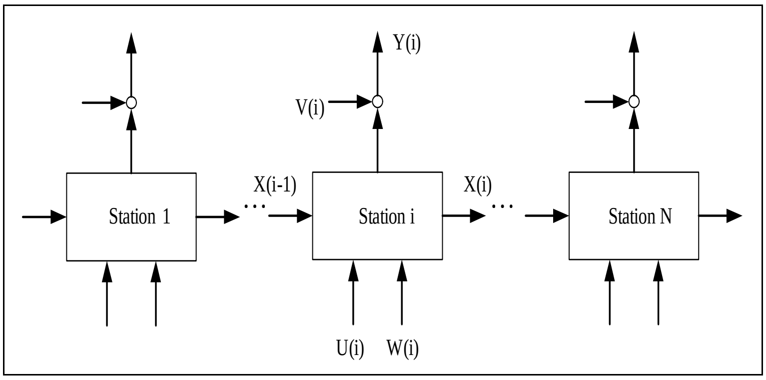

Figure No.1: The stream-of-variation model of N-station assembly process

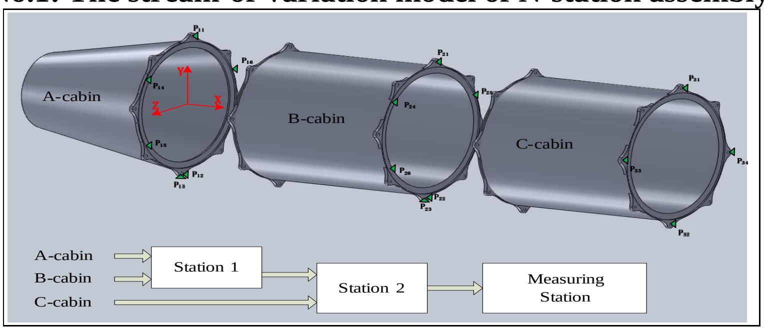

Figure No.2: Assembly process

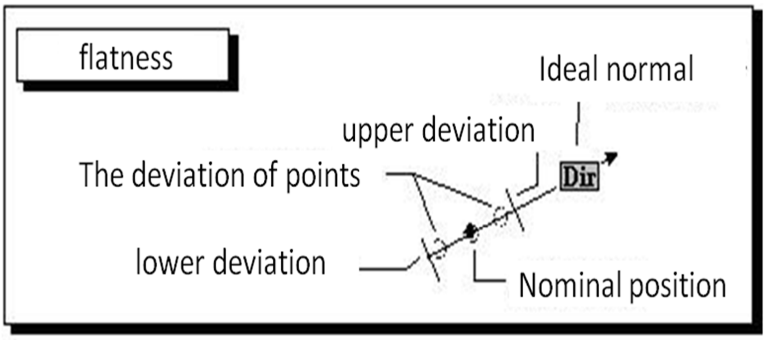

Figure No.3: Flatness

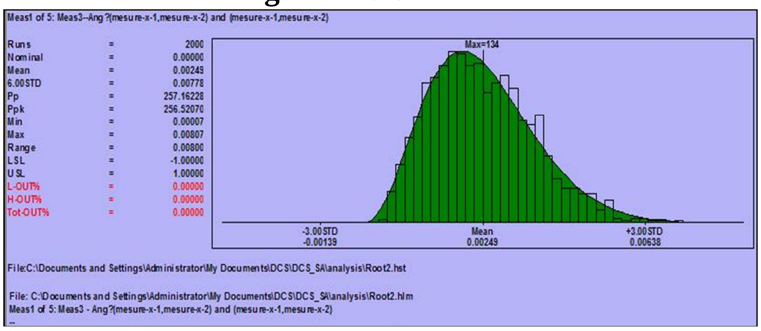

Figure No.4: The angular variation distribution in X axis

CONCLUSION

In this article, the definition of KCC and KPC are proposed at the beginning. And then the assembly sensitivity indices are defined at three different levels according to KCC, namely, the fixture level, the station level and the system level. Based on the state space model of the assembly process, the relationship between Y and U is obtained. Following that, matrix $\gamma^2(i)$ is calculated. So the formulas of different leveled assembly sensitivity are worked out. After that, an aeroplane example is introduced. According to the real assembly process, the theoretical results and the Monte Carlo simulation results are compared, which proved the method in this article is available.

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CONFLICT OF INTEREST

We declare that we have no conflict of interest.

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